Physics 222, Fall 1996
Solutions for Homework Set #8 (due 9/20/96)
Serway, Chapter 21 by Rüdiger Lange

Q1

Problem: Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

Solution: The motion of a simple harmonic oscillator is given by Eqs. [21.1], [21.5] and [21.6] which describe the position, velocity, and acceleration as a function of time. The acceleration follows the equation

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\quad^2 A\cos(\quad t + \quad)$$

The acceleration is not constant but varies sinusoidally in time and becomes zero twice during each cycle of motion.

Q5

Problem: A mass-spring system undergoes simple harmonic motion with an amplitude of A. Does the total energy change if the mass is doubled but the amplitude is not changed? Do the kinetic and the potential energies depend on the mass? Explain.

Solution: The **total energy** of a simple harmonic oscillator is given by $E = \frac{1}{2}kA^2$ and is therefore independent of the mass. If the mass is doubled the energy remains constant.

The **kinetic energy** of this system is

$$E_{KIN} = \frac{1}{2}mv^2 = \frac{1}{2}m(\quad A\cos(\quad t + \quad))^2 = \frac{1}{2}m\quad^2 A^2\cos^2(\quad t + \quad) = \frac{1}{2}kA^2\cos^2(\quad t + \quad)$$

since $\quad = \sqrt{\frac{k}{m}}$.

The **potential energy** can be calculated from $E_{POT} = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\quad t + \quad)$.

We see that the kinetic as well as the potential energy are independent of the mass. The energies are linear functions of the force constant *k* and quadratic functions of the amplitude *A*.

P6

Problem: A 20-g particle moves in simple harmonic motion with a frequency of 3 oscillations/s and an amplitude of 5 cm. (a) Through what total distance does the particle move during one cycle of its motion? (b) What is the maximum speed? Where does it occur? (c) Find the maximum acceleration of the particle. Where in the motion does the maximum acceleration occur?

Solution: (a) Assume that we pull the mass to its maximum displacement at x = 5 cm and release it. The restoring force of the spring will start the harmonic motion and the mass will be accelerated and move through x = 0 to x = -5 cm where it will experience a negative

acceleration from the spring and finally return to the point where we started (x = 5 cm). After one cycle of motion the mass therefore moved a distance of 20 cm.
(b) The maximum speed occurs at x = 0 since the potential energy if zero at this point and all the energy is kinetic energy of the mass. We get the maximum speed if $\sin(\omega t+\varphi)=1$ in Eq. [21.5]. For this case we get $v_{MAX}=\omega A$ where $\omega=2\pi f$ and $f=3\ \mathrm{s}^{-1}$. Therefore $v_{MAX}=2\pi\times 3\ \mathrm{s}^{-1}\times 5\ \mathrm{cm}\ =94.25\ \mathrm{cm/s}$.
(c) The maximum acceleration occurs when the mass reaches the maximum displacement A. Analog to (b) this maximum is given by the amplitude of the osillatory motion in Eq. [21.6].
$a_{MAX}=\omega^2 A=(2\pi\times 3\ \mathrm{s}^{-1})^2\times 5\ \mathrm{cm}=1776.53\ \mathrm{cm/s^2}$.

P13

Problem: A 0.5-kg mass attached to a spring of force constant 8 N/m vibrates with a simple harmonic motion with an amplitude of 10 cm. Caculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the mass is at x = 6 cm from the equilibrium potition, and (c) the time it takes the mass to move from x = 0 to x = 8 cm.
Solution: (a) Like in problem P6 we can determine the maximum velocity and acceleration of the mass from the angular frequency ω and the amplitude A of the motion. First we need to know

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{8\ \mathrm{N/m}}{0.5\ \mathrm{kg}}}=4\ \mathrm{s}^{-1}.$$

In the same way described above we find
$v_{MAX}=4\ \mathrm{s}^{-1}\times 10\ \mathrm{cm}=40\ \mathrm{cm/s}$
$a_{MAX}=(4\ \mathrm{s}^{-1})^2\times 10\ \mathrm{cm}=160\ \mathrm{cm/s^2}$
(b) From the equation of motion $x=A\sin(\omega t)$ we find by solving for t:

$$t=\frac{1}{\omega}\sin^{-1}\left(\frac{x}{A}\right).$$

If we use x = 6 cm and A = 10 cm we find that t = 0.161 s. Now we can use Eqs. [21.5,6] (with $\varphi=0$) to get the speed and acceleration of the mass.

$$v(t=0.161\ \mathrm{s})=4\ \mathrm{s}^{-1}\times 10\ \mathrm{cm}\times\cos(4\ \mathrm{s}^{-1}\times 0.161\ \mathrm{s})=32\ \mathrm{cm/s}$$
$$a(t=0.161\ \mathrm{s})=-(4\ \mathrm{s}^{-1})^2\times 10\ \mathrm{cm}\times\sin(4\ \mathrm{s}^{-1}\times 0.161\ \mathrm{s})=-96\ \mathrm{cm/s^2}.$$

(c) We can calculate the time from the equation of motion as done in part (b). For x = 0 we get t = 0 and for x = 8 cm we get t = 0.23 s. So the time needed to move from the equilibium position to x = 8 cm is 0.23 s.

P41

Problem: An LC circuit consists of a 20-mH inductor and a 0.5-μF capacitor. If the maximum instantaneous current in this circuit is 0.1 A, what is the greatest potential difference that appears across the capacitor?

Solution*:* The energy in an LC circuit is transferred back and forth between the capacitor and the inductor. Therefore the maximum energy stored in the inductor must equal the maximum energy in the capacitor since there are no resistive elements in this curcuit. This means that

$$E_L = \frac{1}{2} L I_{MAX}{}^2 = \frac{1}{2} C V_{MAX}{}^2 = E_C \, .$$

From this we can determine the maximum potential drop across the capacitor.

$$V_{MAX} = \sqrt{\frac{L}{C}} I_{MAX} = \sqrt{\frac{0.02 \text{ H}}{0.5 \times 10^{-6} \text{ F}}} \times 0.1 \text{ A} = 20 \text{ V}$$

P44

Problem: Consider an RLC series circuit consisting of a charged 500-μF capacitor connected to a 32-mH inductor and a resistor R. Calculate the frequency of the oscillations (in Hz) that result for the following values of R: (a) R = 0 (no damping); (b) R = 16 Ω (critical damping: $R = \sqrt{4L/C}$); (c) R = 4 Ω (underdamped: $R < \sqrt{4L/C}$); (d) R = 64 Ω (overdamped: $R > \sqrt{4L/C}$).

Solution: (a) We can determine the frequency of the circuit by Eq. [21.40]. For R = 0 we have an undamped oscillator and the frequency is simply given by

$$= \sqrt{\frac{1}{LC}} = 250 \text{ s}^{-1} \Rightarrow f = 39.8 \text{ Hz}$$

(b) We have critical damping and therefore no oscillations.

(c) With Eq. [21.40] we get $= 242 \text{ s}^{-1} \Rightarrow f = 38.5 \text{ Hz}$

(d) Overdamped oscillator, i.e. no oscillations.